



# A Classical Approach to the De Broglie-Wave Based on Bohr's H-Atom-Model

Thomas Allmendinger

Independent Scholar, Glattbrugg, Zurich, Switzerland

## Email address:

[inventor@sunrise.ch](mailto:inventor@sunrise.ch)

## To cite this article:

Thomas Allmendinger. A Classical Approach to the De Broglie-Wave Based on Bohr's H-Atom-Model. *International Journal of Applied Mathematics and Theoretical Physics*. Vol. 2, No. 1, 2016, pp. 1-15. doi: 10.11648/j.ijamtp.20160201.11

**Received:** June 13, 2016; **Accepted:** June 22, 2016; **Published:** August 1, 2016

---

**Abstract:** The atom model of Niels Bohr is commonly considered as antiquated even if it describes the atomic spectrum of hydrogen quite accurately. The later published relation of Louis De Broglie could arithmetically be implemented into Bohr's formulation, leading to the concept of standing waves as the existence cause of excited electron states. However, at that time it was not possible to find well defined electron trajectories being classically describable. As a consequence, the »quantum mechanics« were developed by several authors, particularly by Schrödinger and by Heisenberg, but delivering a hardly understandable formalism where the classical physical laws appear being abrogated while abstract terms replace concrete and imaginable ones, abandoning the particle perception of mass points. In particular, Heisenberg's »uncertainty principle« and the assumption of state probabilities seem to be in a striking variance to the idea of standing waves. In contrast, a formulation is given here which exactly describes the electron trajectories in the excited states solely by applying classical physical laws. Firstly, the original Bohr-model - in combination with De Broglie's relation - is rolled up. From this formula system a vibration frequency - corresponding to the De-Broglie frequency - is deduced which is n-times larger than the rotation frequency of the Bohr-model. Furthermore, a direct coherence between that vibration-frequency of the electron and the frequency of the involved light is evident being explainable as a resonance effect. Then, a three-dimensional model is proposed where the electron oscillates and pulses perpendicularly to a virtual rotation plane i.e. rotating around a vertical axis, accompanied by a perpetual energy exchange between potential and kinetic energy. This leads at the excited, metastable energy states to well-defined, three-dimensional and wavy trajectories winding up on a surface similar to the one of a hyperboloid, whereas at the ground state the trajectory is planar and stable.

**Keywords:** Quantum-Mechanics, Uncertainty-Principle, Wave-Particle-Dualism, Three-Dimensional Electron-Trajectories, Electron-Oscillation, Resonance-Effect

---

## 1. Introduction: the Historical Background

It may strike as odd that, nowadays – approx. 100 years after its first publication -, somebody else than a historian bothers about Bohr's atom model since quantum mechanics drafted by prominent physicists like *Heisenberg*, *Schrödinger*, *Born*, *Pauli* and *Dirac* were established long ago admitting no further doubts. However: The discussion about quantum mechanics and their roots has not broken off entirely since it is still surrounded by a mist of incomprehensibility, expressed in the bon mot of *Richard P. Feynman* saying that it is impossible to understand quantum mechanics but that it is only possible to get accustomed to it [1]. Therefore, its

genesis has recently been reviewed in several books like this one written by *Manjit Kumar* [2]. As it is well known, the most prominent sceptic was *Albert Einstein*, who had originally made a significant contribution to the atom theory by the discovery and the formulation of the *photoelectric effect* but who could not identify himself with the implicit idea of probability functions instead of definite electron trajectories, being revealed by his famous dictum: „God doesn't dice“. His dispute with Bohr - who just represented the opinion opposing to his own model - culminated in the *Sixth Congress of Solvay* in 1930.

The first important step towards the comprehension of atoms had been made by the discovery of the electron due to the experiments of *J. J. Thomson*, and of *H. Becquerel* with  $\beta$ -radiation being deflected in a magnetic field. Another

important step was made by the study of the electronic spectra of atomic hydrogen - being made at very low temperatures and pressures in the visible and near UV range of light - exhibiting a *discrete line structure*, and in particular the ingenious discovery of *J. J. Balmer* 1885 [3] that the H-spectrum can be described by the following mathematical pattern:

$$\lambda = A \cdot \frac{m^2}{m^2 - n^2} \quad \text{where } \lambda = \text{wave length, } A = \text{empirical constant, } m \text{ and } n \text{ are integers.}$$

Five years later, *J. R. Rydberg* found a more general formulation of this phenomenon, however without being able to explain it. Only after the formulation of *Max Planck's law* in the year 1900, which could explain the spectral dispersion of black radiation by means of the *quantum of action*  $h$ , the inclusion of spectroscopic properties delivered a decisive explanation step. Right after, *Einstein* found 1905 [4] the *photoelectric effect*, relating the energy  $E$  of light with its frequency  $\nu$  over *Planck's constant*  $h$ :

$$E = h \cdot \nu$$

After the insufficient model proposal of *Thomson* and the more illuminating one of *Rutherford* in the year 1911, *Niels Bohr* published in the year 1914 a model for the hydrogen atom being able to widely explain its spectral properties [5]. It was based on the hypothesis that the electron rotates around the proton at different definite radii (see Fig. 1), corresponding to different energy levels whereby *its angular momentum is quantified* exhibiting the value of  $h/2\pi$  at the ground state and an integer multiple  $n$  at the excited states, and that possible jumps between these energy levels are linked to the adsorption or emission of light and to the frequencies given by Einstein's relation. As a further consequence, the circular current of the electron induces a *magnetic moment* of the atom which can be measured experimentally.

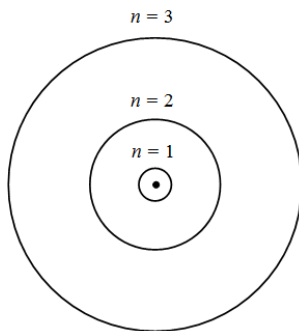


Fig. 1. Relative electron radii within the Bohr-model.

The main argument against the Bohr model was and still is that the H-atom could be compared with a so called *Hertz dipole*, thus emitting electromagnetic radiation till a collapse occurs while the electron falls onto the nucleus. This provokes the question whether the electromagnetic radiation, proceeding with frequencies  $\geq 10^5$  Hz (and in the case of the H-atomic radiation even of about  $10^{15}$  Hz), may be reduced

on a rotating macro-physical barbell-like dipole where - e.g. with an ultracentrifuge - no frequencies larger than  $10^4$  Hz are achievable which makes a verification impossible.

Furthermore, the Bohr model exhibits quite a lot of obvious deficiencies with regard to empirical facts. So it isn't able to explain the *fine-structure* of atomic spectres appearing when a magnetic field is applied. (Hence, *Sommerfeld* [6] extended Bohr's model assuming an additional directional quantification leading to *elliptical orbits*, but that explanation was not sufficient for any case.) Moreover, the model fails for describing more complex atoms, and much less for chemical bonds or for the periodic table of the elements. And, even more basically, it could not explain the fact that the angular momentum is quantified. But ten years after that, *Louis De Broglie* found in his thesis 1924 an answer to the question why solely definite energy levels are possible, determining a discrete spectrum. He assumed that the electron moves on a *wavy* trajectory around the nucleus, and that solely trajectories representing *standing waves* are allowed. Thereby, De Broglie's equation concerning the relation between wavelength and momentum ( $p_{el} = h/\lambda_{el}$ ) may be derived from Bohr's quantum relation - or vice versa -, so it is compatible with it, delivering the - here called - *Bohr-De Broglie (BDB) model*.

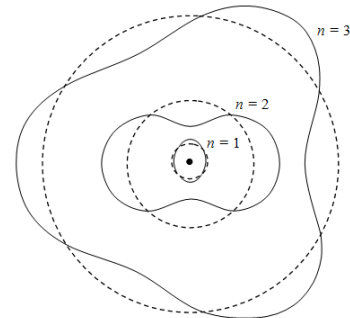


Fig. 2. Alleged perception the of the electronic wave, according to [9].

The phenomenon of the wave nature of the electron has been empirically verified by numerous experiments using electron beams being accelerated by electric fields, in particular observing diffraction on thin metal foils [7]. However, no explanation could be found for the *cause* of this wavy movement. Rather it was assumed that the electron would undulate quasi by nature. Philosophers even used to speak of a mysterious »wave-particle-dualism«. Above all, it seemed not possible to precisely calculate the wavy electron trajectories. Therefore, in all usual textbooks any descriptive illustration of the electron's wavy trajectory is missing. Such an illustration could be solely found in a quite esoterically appearing book, however being not at all convincing (Fig. 2). A similar »qualitative« illustration is also given in the textbook of *G. Herzberg* [8].

As a consequence of these deficiencies, an alternative approach was formulated, starting from De Broglie's wave concept. Thereby, *Werner Heisenberg* [10] developed a matrix calculation, while *Erwin Schrödinger* [11] [12] [13] established a differential equation on the basis of Cartesian

coordinates using the *Hamiltonian*, delivering well determined *Eigenvalues* for the energy states but not well defined electron trajectories. Instead of those, the *squares* of the wave functions were given, representing – according to *Max Born's* interpretation [14] [15] – the *probabilities of presence of electrons* and *abandoning the principle of determinism*. Therein, initially *three independent quantum numbers* were provided, namely the *main quantum number n*, defining the shells K, L, M etc., the *auxiliary or angular momentum quantum number l*, being responsible for the special direction and named as s-, p-, d- and f-states, and finally the *magnetic quantum number m<sub>l</sub>* being related to the magnetic moment induced by the electron orbit.

However, some principle objections against this theory seem to be appropriate: First of all, Schrödinger's citation of the *Hamiltonian*, being based on the *Lagrange-function*, appears a priori devious since that theory affects *extended systems of mass points*, as they are relevant particularly in thermodynamics, and *not* an isolated two-particle system which is given by the hydrogen atom. But the most striking feature of this theory was the modification of De Broglie's wave by *Heisenberg's »indeterminacy-principle«*, according to which location and momentum of a particle are not simultaneously determinable, namely

De Broglie:  $\lambda_{el} \cdot p_{el} = h$  with  $\lambda_{el}$  = electron wave length, and  $p_{el}$  = momentum

Heisenberg:  $\Delta x \cdot \Delta p_x \geq h$  where  $\Delta x$  stands for the indeterminacy of the location.

Bohr, denying his own original model, made an ulterior step assuming the position of the electron being *really* uncertain, which subsequently led to the idea of electron »orbitals«, sometimes being compared with electron-clouds, and showing at least two significant differences to the BDB-model: Firstly, within that model, the s-orbital of the ground state exhibits *no* angular momentum ( $l=0$ ) and is *spherically symmetric*, while the trajectory in Bohr's model is *circular* and *planar*, exhibiting a basic angular momentum. Thus, in the former case, the hydrogen-atom appears as a ball, whereas, in the latter case, it's like a disc, or – in respect of the electron – a ring. The formulation implied quasi a trick overriding the assumed Hertz's dipole radiation. And secondly, the conclusion seems rather paradoxical that a well-defined electron trajectory should not really exist though the existence of a standing electron wave is assumed. Hence, the plausible perception of a standing wave as a governing principle loses its fundament.

Further aspects concern the spectra being recorded in the presence of a *magnetic field*, inducing shifts and widening of lines which *Pieter Zeeman* had observed and published already in the year 1897. Moreover, in some cases the multiplicity of certain lines turned out to be larger than expected, that phenomenon being denoted as *anomalous Zeeman-effect*. As a consequence, but only in the year 1925 *Goudsmith* and *Uhlenbeck* suggested that the electron exhibits an own angular momentum being incident to a magnetic moment which subsequently was denoted from

then on as »*spin*«. Hence the electron-spin needed the introduction of a *fourth quantum number s* into the theory. Moreover, it had to be implemented into the convenient theory which was realized mainly by *Pauli* and *Dirac*, the latter one applying a relativistic matrix calculation. These special effects are not within the scope of the present treatise, except the fact that the spin momentum of the electron is equal to its angular momentum in the ground state of the BDB-model, which cannot be reasonably explained by the conventional theory.

Hence, in spite of the several further experimental methods which have been introduced and improved since then, and in spite of the theoretical refinement made by the *QED-theory (Quantum Electro Dynamics)*, the fundament of the wave mechanics, established not in one go, and not by a single but by several authors, has never been questioned whereas the original Bohr-model has been abandoned. Solely in the 1950s *Bohm* formulated a deterministic theory regarding De Broglie's hypothesis and assuming a »quantum theory without observers« but also using differential equations and also being quite abstract. This approach was persisted by *D. Dürr* and *S. Teufel* [16], but it doesn't correspond to the here proposed approach and will not be discussed hereafter.

Nevertheless, the purpose of the present consideration is going back to the roots and improving the original BDB model of hydrogen by introducing a *third dimension* and by *implementing an additional oscillating motion* of the electron, regarding *the perpetual exchange of potential and kinetic energy* as it is the characteristic requirement for any oscillation. Moreover, a connection between the electron spin and the electron wave may be assumed, involving the conclusion that the real origin of the spin is not a rotation – as the name »spin« suggests – but an oscillation.

## 2. The Usual Derivation of the BDB-Model for the H-Atom

According to Meschede's revision of Gerthsen's basic German physical text book [17], the Bohr model „is the last model which also a beginner really can understand“. Hereinafter, initially the original version according to Bohr shall be described in simplified terms, whereon the De Broglie hypothesis shall be introduced (delivering the *BDB = Bohr De Broglie-model*). It has to be said in advance that the gravitation force can be neglected with respect to the Coulomb force; that furthermore the nucleus proton may be considered as stationary; that the electron orbit may be assumed to be circular; and that any relativistic effects may be neglected. To avoid the confusion with the *frequency abbreviation v*, for the *velocity* the symbol *u* is used. Apart from that, the following abbreviations and units are used:

mass *m* [kg]

time *t* [s]

period *T* [s]

wave length  $\lambda$  [m]

angular velocity  $\omega$ : radian measure [s<sup>-1</sup>] with  $2\pi \div 360^\circ$

momentum  $p$  [kgms<sup>-1</sup>]

angular momentum  $L$  [kgm<sup>2</sup>s<sup>-1</sup>] = [Js]

force  $F$  [1 N = 1 kgms<sup>-2</sup>]

work = energy  $E$  [J] = [kgm<sup>2</sup>s<sup>-2</sup>] = [Ws]

For the calculations, the following values are assumed:

dielectric constant  $\epsilon_0 = 8.854187817 \cdot 10^{-12}$  AsV<sup>-1</sup>m<sup>-1</sup>

elementary charge  $e = 1.60217646 \cdot 10^{-19}$  C

Planck's constant  $h = 6.6260688 \cdot 10^{-34}$  Js

rest mass of the electron  $m_e = 9.1093819 \cdot 10^{-31}$  kg

First of all, the equilibrium condition between the centrifugal force and the Coulomb attraction has to be regarded:

$$\frac{m_e u^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \quad \text{. That may be rearranged to}$$

$$u^2 r = \frac{e^2}{4\pi\epsilon_0 m_e} \quad (1)$$

Secondly, the quantization condition for the *angular momentum*  $L$  has to be formulated:

$$|L_n| = m_e \cdot u_n \cdot r_n = \frac{n \cdot h}{2\pi} \quad (2)$$

This condition comprises the so-called *principal quantum number*  $n$ . It represents the basic relation of quantum mechanics and interrelates to the already mentioned and subsequently considered photoelectric effect being discovered by Einstein.

The combination of these equations delivers an expression for the (*rotation-*) *velocity*

$$u_n = \frac{e^2}{2n\epsilon_0 h} \quad (3)$$

or

$$u_n = \frac{u_1}{n} \quad (4)$$

$$\text{with } u_1 = 2.1877 \cdot 10^6 \text{ ms}^{-1} \approx c_{\text{light}} / 137$$

as well as for the *atomic radius* being equivalent to the radius of the electron orbit:

$$r_n = \frac{n^2 \epsilon_0 h^2}{\pi m_e e^2} \quad (5)$$

or

$$r_n = r_1 \cdot n^2 \quad (6)$$

$$\text{with } r_1 = 0.5292 \cdot 10^{-10} \text{ m}$$

Since the angular momentum is defined as the product of mass, velocity and radius, an expression equivalent to (2) is got when these relations are inserted in (2):

$$|L_n| = m_e \cdot u_n \cdot r_n = m_e \cdot u_1 \cdot r_1 \cdot n = \frac{n \cdot h}{2\pi} \quad (7)$$

When  $n = 1$ , the ground state is attained, being characterized by a maximum velocity, a minimum radius and a minimum angular momentum.

Now, a connection between the difference of two energy levels and the frequency of the absorbed or emitted light should be established by means of the Einstein equation

$$\Delta E = h\nu_{\text{light}} \quad (8)$$

For this purpose, the aggregate amount of energy – i.e. the *total energy*, comprising *potential* and *kinetic* energy - has to be calculated for any level. Thereby, a negative sign has to be attributed to the potential energy, that one becoming zero when the radius is infinite and becoming negative when the radius is getting smaller:

$$E_{\text{pot}} = \frac{-e^2}{4\pi\epsilon_0 r} = -m_e u^2 \quad (9)$$

$$E_{\text{kin}} = \frac{1}{2} m_e u^2 \quad (10)$$

$$E_{\text{tot}} = E_{\text{pot}} + E_{\text{kin}} = -\frac{1}{2} m_e u^2 = -\frac{e^2}{8\pi\epsilon_0 r} \quad (11)$$

When the quantised radius according to (5) is inserted in equation (11), the total energy can be expressed by  $h$ :

$$E_{\text{tot}} = -\frac{m_e e^4}{8n^2 \epsilon_0^2 h^2} \quad (12)$$

Finally, its combination with equation (8) yields equation (13):

$$\nu_{\text{light}} = \frac{m_e e^4}{8\epsilon_0^2 h^3} \left( \frac{1}{n_x^2} - \frac{1}{n_y^2} \right) \quad (13)$$

with the conditions  $n_y > n_x$  and  $n_x \geq 1$ .

The thus calculated values fit the frequencies of the hydrogen spectrum quite well. The term ahead the bracket is named *Rydberg constant*; its value is  $3.2899 \cdot 10^{15} \text{ s}^{-1}$ .

Additionally, the *De Broglie hypothesis of the standing waves* shall be regarded. The condition of a standing wave may be expressed by the formula

$$n \cdot \lambda_{\text{electron}} = 2\pi \cdot r_n \quad \text{or rearranged} \quad r_n = \frac{n \cdot \lambda_{\text{electron}}}{2\pi} \quad (14)$$

When this relation for  $r_n$  is inserted into formula (2) expressing the angular momentum

$$m_e \cdot u_n \cdot r_n = \frac{n \cdot h}{2\pi} = m_e \cdot u_n \cdot \frac{n \cdot \lambda_{\text{electron}}}{2\pi},$$

the result reveals nothing other than the De Broglie relation (15):

$$m_e \cdot u_n = p_e = h/\lambda_{\text{electron}} \quad (15)$$

Regarding this, it must be assumed that not only a wave length but also a *wave frequency* is assigned to the De Broglie wave. Consequently, the relation between this wave frequency and the rotation frequency of the electron shall be determined.

### 3. The Vibratory-Like Frequency of the Electron in the BDB-Model

As for every wavy motion, the condition (16) must be fulfilled comprising the velocity  $u_{rot}$  as the product of the vibratory-like frequency  $\nu_{vib}$  and the wave length  $\lambda_{electron}$ :

$$u_{rot} = u_n = \lambda_{electron} \nu_{vib} \quad (16)$$

Thus the electron motion is characterized by *two different frequencies*: the *rotation frequency* and the *vibratory-like frequency*, the latter one being part of the former one:

$$\nu_{vib} = u_{rot} / \lambda_{electron} \quad (17)$$

Furthermore, it may be defined

$$\nu_{rot} = u_{rot} / 2\pi r_n \quad (18)$$

The combination of the equations (17) and (18) yields

$$\nu_{vib} = n \nu_{rot} / 2\pi r_n \quad (19)$$

And finally, when relation (19) is compared with equation (18), the quite interesting relation (20) is gotten:

$$\nu_{vib} = n \nu_{rot} \quad (20)$$

Equation (20) amounts that the two frequencies are coupled each to the other via the quantum number  $n$ , being equal when  $n = 1$ .

### 4. The Coherence between Electron-Frequency and Light-Frequency

Now it shall be determined how the electron-frequency and the light-frequency are related. Hereto,  $\nu_{vib}$  and  $\nu_{rot}$  have to be expressed by  $\epsilon_0$ ,  $e$ ,  $h$  and  $n$ , that which is feasible by combining the equations (3), (15), (16), (17) and (20):

$$\nu_{vib} = \frac{m_e e^4}{4n^2 \epsilon_0^2 h^3} \quad (21)$$

$$\text{resp. } \nu_{rot} = \frac{m_e e^4}{4n^3 \epsilon_0^2 h^3} \quad (22)$$

If the power of relation of  $n$  is regarded, it is evident that among these two equations only the first one, namely (21), is compatible with relation (13) being relevant for the light frequency. This means that the *light frequency is directly*

*correlated to the vibration frequency of the electron and not to its rotation frequency* (except when  $n = 1$ , i.e. when both frequencies are equal). Equation (21) may be rearranged to equation (23):

$$\frac{1}{n^2} = \nu_{vib} \frac{4\epsilon_0^2 h^3}{m_e e^4} \quad (23)$$

That one may be inserted in equation (13) and thus delivering the relation (24):

$$\nu_{light} = \frac{1}{2} (\nu_{vib1} - \nu_{vib2}) \quad (24)$$

In words: The frequency of the absorbed or emitted light corresponds to the half of the difference of the vibration frequencies of the electron being expected for the relevant orbits. The factor of  $\frac{1}{2}$  seems strange and is not simply explicable. However, the statement is decisive that a direct proportionality exists between the light frequency and the vibration frequency of the electron. Therefore, it is standing to reason how these vibrational electron trajectories really appear. However, before studying this question, the Hertz-oscillator objection must be considered.

### 5. The Comparison with the Hertz-Oscillator

Sometimes the electron which circles around the nucleus is compared with the so called *Hertz-oscillator*, being imagined as a *rotating dipole* and thus radiating electromagnetically. That model-concept should explain the light emission of the (hydrogen-) atom being in an excited state. Thereby it is *not* explainable why the electron does not finally bear down on the nucleus while it emits further electromagnetic radiation but, instead of that, remaining on a minimal orbit which cannot be exceeded below.

For the *radiation-power*  $P$  (i.e. for the emitted energy per time) usually the following formula is given, deduced by means of the *Maxwell-Laws* and usually found in textbooks:

$$P = - \frac{p_d^2 \omega^4}{6\pi \epsilon_0 c^3} \quad \text{unit: [W]} \quad (25)$$

where  $p_d$  means the dipole-momentum (27).

In that formula the angular velocity may be expressed by the frequency according to the relation  $\omega = 2\pi\nu$  so that the equation may be written as

$$P = - \frac{8\pi^3 p_d^2 \nu^4}{3\epsilon_0 c^3} \quad (26)$$

In addition, the relation for the dipole-momentum has to be regarded:

$$p_d = Q \cdot r \quad \text{where } Q = \text{amount of charge} \quad (27)$$

Delivering to the radiation-power the relation:

$$P = -\frac{8\pi^3 Q^2 r^2 \nu^4}{3\epsilon_0 c^3} \quad (28)$$

For getting the radiation energy (per cycle), one has to divide by the frequency  $\nu$ :

$$E = -\frac{8\pi^3 Q^2 r^2 \nu^3}{3\epsilon_0 c^3} \quad (29)$$

unit: [Ws]

But at least now the attempt finding an analogy to the relations being valid within the Bohr-model should be abandoned, considering the following reasons:

1. Within this classical electrodynamic model (CEDM),  $Q$ ,  $r$  and  $\nu$  are arbitrary and thus independent of each other. Within the Bohr-model, that's not the case, being already obvious from equation (1).
2. Within the CEDM, a dependency of the radiation energy on the *rotation* frequency is predetermined – moreover in the cube –, whilst within the Bohr-model a proportionality to the *vibration* frequency exists. The latter one does not at all appear within the CEDM.
3. Within the CEDM, the emission is assumed occurring equally in all directions, that which is probably not the case within the Bohr-model when a singular atom is considered.
4. The Hertz-model originally relates to a *spark inductor* and *not* to a *rotating dipole* as it would be the case e.g. at a rotating condenser. Thereby, the electrons move into metallic conductors exhibiting other conditions than in the case of electrons in the absence of such conductors. A concrete experiment which correctly models the conditions prevailing within an H-atom is not known.

Therefore, an application of the hitherto customary model of the Hertz-oscillator onto the Bohr-model seems being not admissible.

## 6. The Three-Dimensional Modification of the BDB-Model

From the above follows that the BDB-model implies - besides the rotation - an additional vibration of the electron whose frequency is  $n$ -times as much as the rotation frequency, but it cannot outline it figuratively. For eradicating this flaw it seems obvious to use a *three-dimensional description* by classical means supposing a three-dimensional electron trajectory since the two-dimensional feasibilities are exhausted due to the quantum condition of the angular momentum. Thereby, the electron should perform an *undulation* while it orbits the nucleus, this wave being obliged to be a *standing one*, according to the De Broglie condition. Furthermore, that undulation may be regarded as the combination of a *horizontal rotation*, on the one side, and of a *vertical vibration*, occurring at least as an *oscillation*, on the other side. This oscillation is due to a *perpetual exchange of potential and kinetic energy*, as it is characteristic for any oscillation.

An obvious approach for expressing these properties is a formal one, applying a differential equation which reveals that the decrease of the potential energy must be equal to the increase of the kinetic energy while the total energy remains constant, i.e.

$$\frac{\partial E_{pot}}{\partial t} = -\frac{\partial E_{kin}}{\partial t} \quad (30)$$

That yields, using equation (9), the relation

$$\frac{e^2}{4\pi\epsilon_0} \cdot \frac{\dot{r}}{r^2} = -m_e u \cdot \dot{u} \quad (31)$$

However, the complex character of the whole process, comprising much more than a simple oscillator and involving additionally at least a rotation, entails that this expression actually comprises so many unknowns that it gets impractical. In particular, the temporal course of the radius  $r$  as well as that one of the velocity  $u$  is not known, the latter one being composed of different kinds of velocities whose correlations are not simply evident. So it seems necessary starting from a vivid model implying some plausible assumptions and applying physically correct relations which must lead to consistent results. A considerable handicap is thereby the fact that it is not possible to construct an adequate macro physical model. The unique empirical proof is given by the discrete spectres, while the correlation to the Bohr model emerges from the assumption that it solely represents a *limiting case* of the vibration or oscillation process in the atom, namely characterized by *the top position of the electron where the oscillation rate is zero*. Thereby, the same simplifications as within Bohr's model shall be made, i.e. the nucleus is assumed as stationary, and the gravitation force is neglected.

A basic premise at the here applied model is entailed in the assumption that the energetic excitation of the electron may be conceived as the result of a *lateral electromagnetic impact* being perpendicular to its virtual rotation plane leading in the excited electronic energy states to three-dimensional trajectories winding up on a three dimensional surface whereas in the ground state the trajectory is planar. Since that excitation takes place perpendicularly to the rotation plane, *the orbital angular momentum of the ground state must remain constant also in the excited states*, according to the theorem of constant orbital momentum. This assumption is of great importance for the whole theory. At first it seems peculiar considering the fact that, according to equation (2), the Bohr model implies an  $n$ -fold magnitude of the orbital momentum in the excited states. However, that condition only concerns the *top position* of the electron, corresponding to Bohr's model, while the behavior of the total orbital momentum during the vibration process is quite complex. For avoiding a confusion with Bohr's rotation velocity  $u_n$  as well as Bohr's radius  $r_n$ , the terms  $u_{rot}^*$  and  $r_{rot}^*$  are introduced, both being time-dependent and – still – dependent on  $n$ . Therefore, the following relation shall be valid:

$$u_{rot}^* \cdot r_{rot}^* = u_1 \cdot r_1 \quad (32)$$

where  $u_1$  and  $r_1$  represent Bohr's terms for the ground state.

Since during the process the oscillation velocity - and thus the relevant kinetic energy - is varying, but since simultaneously the potential energy is also varying due to the distance variation between electron and nucleus, it is in principle thinkable that the difference of the kinetic oscillation energy is exactly equal to the difference of the potential energy. As a consequence, the kinetic rotation energy - and implicitly the rotation velocity - would remain constant meaning that the three-dimensional trajectories of the electron in the excited states would wind up on a cylindrical surface.

However, as the analytical examination has yielded, this isn't really the case. Instead, the electron trajectories resemble to *hyperboloids*, which means that an additional motion, here called »pulsation«, must exist delivering a further kinetic energy input, and engendering a variation of the rotation velocity. Therefore, the »vibration« is composed of two components: of a vertical *oscillation* and of a horizontal *pulsation*, the former being vertical to the rotation plane, and the latter being parallel to the rotation plane.

For being able to describe this process, it is necessary to choose a system with *polar coordinates*, combined with *Cartesian coordinates* allowing differing illustrations, depending on the objectives. Using this, three prominent states may be outlined: the limiting case of the top position where  $u_{osc}$  and  $u_{puls}$  are zero, being equivalent to the conditions of the Bohr model (Fig. 3a); the general case of the intermediate state where the process is in full swing (Fig. 3b); and the limiting case at the equator where  $u_{osc}$  is maximal and  $u_{puls}$  is zero (Fig. 3c). Therein,  $d$  is the abbreviation for the deflection, while the deflection angle is denoted as  $\varphi$ . The abbreviations within the indices: *rot* for »rotation«, *osc* for »oscillation«, *puls* for »pulsation«, and *eq* for »equator«, while the indication *top* is used when the electron is located at the maximal position. The rotation angle  $\psi^*$  being employed below for computations, is associated to  $u_{rot}$  but not mapped in these figures. The same is true for the rotation angle  $\psi$  being associated to the Bohr model. The radius  $r^*$  is provided with an asterisk to make obvious that it is time-dependant, and not constant like the Bohr radius  $r_n$ .

## 7. Basic Assumptions and Model Calculations

For being able to mathematically conceive this complex process, a stepwise proceeding is necessary, starting from the definite limiting conditions being apparent in the Figures 3a and 3c. Thereto it is necessary to formulate basic assumptions or physical theorems according with natural laws but being adapted to that special constellation.

One of these assumptions has already been mentioned, namely the *law of conservation of the angular momentum*, expressed with equation (32). A further but trivial assumption

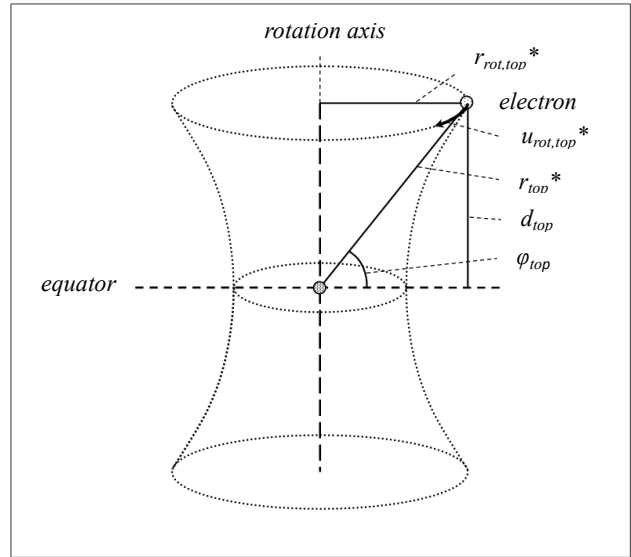


Fig. 3a. Model draft of the limiting case at the top position of the electron.

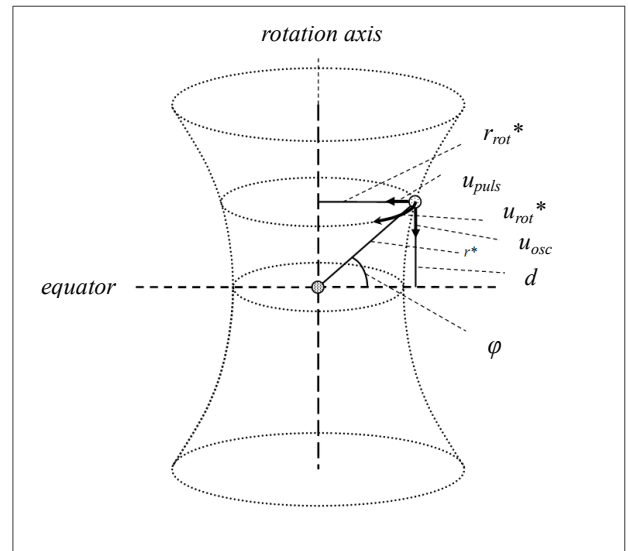


Fig. 3b. Model draft of the intermediate state.

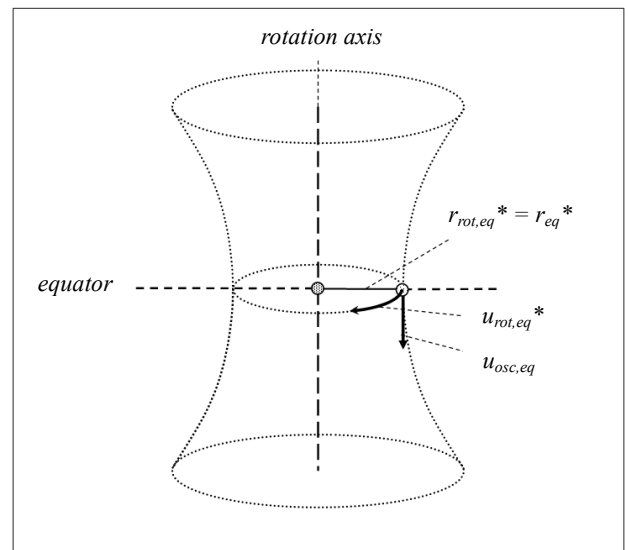


Fig. 3c. Model draft of the limiting case at the equator.



is given by the *theorem of Pythagoras*

$$(r_{rot}^*)^2 + d^2 = (r^*)^2 \tag{33}$$

Of course, the *law of conservation of energy* has to be considered, too. Thereby, *the total energy at a certain energy level according to equation (11) of the Bohr model* is relevant, being advantageously expressed in terms of  $u_1$  and  $r_1$  given in the equations (3), (4), (5) and (6):

$$\begin{aligned} E_{tot} = E_{pot} + E_{kin} &\rightarrow -\frac{e^2}{8\pi\epsilon_0 r_n} = -\frac{e^2}{4\pi\epsilon_0 r^*} + \frac{1}{2} m_e \cdot (u_{tot})^2 \\ &\rightarrow \frac{2r_1}{r^*} = \frac{1}{n^2} + \frac{(u_{tot})^2}{u_1^2} \end{aligned} \tag{34}$$

Within the *first limiting case*, i.e. at the top position of the electron, the vibration terms disappear, which leads to the following simplifications:

$$r_{top}^* = r_n = r_1 \cdot n^2 \tag{35}$$

and

$$u_{tot,top} = u_{rot,top}^* = u_n = \frac{u_1}{n} \tag{36}$$

As a consequence, based on (32), the values of  $r_{rot,top}^*$  can be determined:

$$r_{rot,top}^* = r_1 \cdot \frac{u_1}{u_{rot,top}^*} = r_1 \cdot n \tag{37}$$

hence yielding for  $n = 2$  the value of  $2 r_1$ , and for  $n = 3$  the value of  $3 r_1$ , etc.

The knowledge of  $r_{top}^*$  and of  $r_{rot,top}^*$  enables the determination of  $\phi_{top}$  due to the relation

$$\cos \phi_{top} = \frac{r_{rot,top}^*}{r_{top}^*} = \frac{1}{n} \tag{38}$$

yielding the values  $\phi_{top,2} = 60^\circ$  and  $\phi_{top,3} = 70.53^\circ$ .

On the other hand, since

$$\sin \phi_{top} = \frac{d_{top}}{r_{top}^*} = \frac{d_{top}}{r_1 \cdot n^2} \tag{39}$$

$d_{top}$  can be determined, too, yielding the values  $3.4641 r_1$  and  $8.4853 r_1$  for  $n = 2$  and  $n = 3$ . (For comparison: The respective values of  $r_n$  are  $4 r_1$  and  $9 r_1$ ).

Within the *second limiting case*, i.e. at the equatorial position of the electron, the pulsation gets zero having the following consequences onto the terms:

$$r_{rot,eq}^* = r_{eq}^* \tag{40}$$

and

$$(u_{tot})^2 = (u_{rot,eq}^*)^2 + (u_{puls,eq})^2 \tag{41}$$

However, for making the formula system resolvable, an additional assumption has to be made, known as the *equipartition law of (kinetic) energy*:

$$u_{rot,eq}^* = u_{puls,eq} \tag{42}$$

and

$$(u_{tot})^2 = 2(u_{rot,eq}^*)^2 \tag{43}$$

It has to be mentioned that this »law« cannot be generally valid for the whole process since e.g.  $u_{puls}$  disappears at the top position of the electron. Yet, a generalization will be suggested later.

Then, considering these relations,  $u_{rot,eq}^*$  and  $r_{rot,eq}^*$  can be determined:

Firstly, the relations (34) and (43) deliver the equation

$$\frac{2r_1}{r_{rot,eq}^*} = \frac{1}{n^2} + \frac{2(u_{rot,eq}^*)^2}{u_1^2}$$

Then, the implementation of (32) yields the equation

$$\frac{2u_{rot,eq}^*}{u_1} = \frac{1}{n^2} + \frac{2(u_{rot,eq}^*)^2}{u_1^2}$$

And finally, the viable solution of this quadratic equation is

$$\frac{u_{rot,eq}^*}{u_1} = \frac{1}{2} + \frac{\sqrt{n^2 - 2}}{2n} \tag{44}$$

For  $n = 1$ , the result is imaginary and indeed non-existent. This is plausible since it concerns the ground state which cannot be subject of this procedure. However, for the excited states with  $n < 1$ , real results for the equatorial rotation velocities are obtained, moreover delivering the corresponding values of the radii, due to equation (32):

$n$	$u_{rot,eq}^*/u_1$	$r_{rot,eq}^*/r_1$
2	0.85355	1.17158
3	0.94406	1.05925

Summarizing, up to now the following values have been evaluated:

**Tab. 1.** Calculated values as a function of  $n$

$n$	$r_n$	$r_{rot,top}^*$	$r_{rot,eq}^*$	$d_{top}$	$\phi_{top}$	$u_n$	$u_{rot,top}^*$	$u_{rot,eq}^*$
2	4	2	1.17	3.46	60.0°	0.50	0.50	0.85
3	9	3	1.06	8.49	70.5°	0.33	0.33	0.96

(The units for radii are given in multiples of  $r_1$ , and those for rates in multiples of  $u_1$ )



After having now clarified the situations at the two limiting cases, the more difficult question of the intermediate states arises, particularly of the temporal course of the deflection  $d$ . Hereto, a time base is required. Since this model is based on Bohr's model, it immediately suggests itself to assume that the Bohr-rotation, i.e. its angular velocity  $\omega_n$ , dictates the rhythm of the whole process. Thereby, the application of a harmonic oscillator seems to be adequate even if the situation is not exactly equal to that of a spring pendulum where the relevant sinus-function can be strictly deduced from a differential equation. Hence, the following approach is made:

$$d = d_{top} \cdot \sin(n \cdot \omega_n \cdot t) = d_{top} \cdot \sin(n \cdot \psi) \quad (45)$$

with  $\omega_n = \frac{u_n}{r_n} = \frac{u_1}{r_1 \cdot n^3}$

$$\dot{d} = u_{osc} = n \cdot \omega_n \cdot d_{top} \cdot \cos(n \cdot \psi) = \frac{u_1}{r_1 \cdot n^2} \cdot d_{top} \cdot \cos(n \cdot \psi) \quad (46)$$

At the equator, the cosine becomes 1. Therefore, equation (46) turns into

$$\dot{d}_{eq} = u_{osc,eq} = \frac{u_1}{r_1 \cdot n^2} \cdot d_{top} \quad (47)$$

Since, according to equation (42), the oscillation velocity is equal to the rotation velocity,  $d_{top}$  may be calculated alternatively, using the harmonic oscillator approach (47) and the values for the rotation velocity which have already been gotten due to equation (43). These values may be compared with the previously determined values of  $d_{top}$  according to equation (39), that comparison delivering a grade for the approximate validity of the harmonic oscillator approach:

n	$d_{top}$ (directly)	$d_{top}$ (HO)	accordance
2	3.4641 $r_1$	3.4142 $r_1$	98.6 %
3	8.4853 $r_1$	8.4965 $r_1$	100.1 %

$d_{top}$  (directly) means: determined directly

$d_{top}$  (HO) means: determined via harmonic oscillator

Due to the good accordance of the results using the two calculation-proceedings, the plausibility of the approach is well confirmed. However, it also concedes that the harmonic oscillator solely represents an approximation of the real process. That's not striking considering the bended shape of the oscillating path. In view of the complex conditions, the exact mathematical derivation might be very difficult, if at all possible.

The next step consists in finding regularity for the intermediate states, i.e. for the courses of the trajectories, preferably as a function of  $\psi$ , between the two limiting cases »top position« and »equator«. Hereof, the plot of  $u_{osc}$  vs  $d$  is unproblematic since for both terms the harmonic oscillator relations (45) and (46) may be used, delivering the diagrams in the Fig. 4a and 4b.

The determination of  $r_{rot}^*$  is much more delicate. Since a

logical derivation seems not feasible, the following arbitrary but plausible assumption has been made, fulfilling the conditions at the limiting cases and delivering the key for its calculation:

$$r_{rot}^* = r_{rot,eq}^* + (r_{rot,top}^* - r_{rot,eq}^*) \cdot (\sin(n \cdot \psi))^2 \quad (48)$$

The respective figures 5a and 5b – where  $d$  is determined by the harmonic oscillator relation (45) - reveal the bended character of the oscillation resembling to a hyperboloid. If instead of the square the single of the sine is inserted, the resulting curve becomes nearly linear exhibiting a kink at the equator, i.e. being V-shaped, and thus appearing not plausible.

Besides, formula (48) enables the determination of the pulsation rate being the temporal derivation of  $r_{rot}^*$ , delivering the following relation:

$$u_{puls} = \dot{r}_{rot}^* = \frac{2u_1}{r_1 \cdot n^2} (r_{rot,top}^* - r_{rot,eq}^*) \cdot \sin(n \cdot \psi) \cdot \cos(n \cdot \psi) \quad (49)$$

The respective plots ( $u_{puls}$  vs  $d$ ) are shown in Fig. 6a and 6b. Compared to  $u_{osc}$  outlined in Fig. 4,  $u_{puls}$  is generally minor, being well intelligible in view of the shorter distances.

The last hurdle to clear is finding a connection between the »Bohr-angle«  $\psi$  and the angle  $\psi^*$ , the latter one being operant within the equator plane and being a promising parameter for describing this process, delivering the Figures 7 - 10. There-to, the fact may be adduced that the temporal periods of both processes must be equal, considering the circumstance that the »Bohr rotation« according to  $\psi$  is – within the excited states – constant but only virtual, whereas the real rotation, according to  $\psi^*$ , is not constant but dependent on time. The relevant presumption for getting such a relation is

$$\int_0^{T_n} \omega^* \partial t = \int_0^{T_n} \frac{u_{rot}^*}{r_{rot}^*} \partial t = 2\pi \quad (50)$$

where  $r_{rot}^*$  is given by the relation (48), and  $u_{rot}^*$  can be calculated by equation (32). However, since this integral appears difficult to solve explicitly, an infinitesimal summing up procedure may be applied as a viable alternative, using the relation

$$\psi^* = \sum_0^{T_n} \Delta \frac{u_{rot}^*}{r_{rot}^*} \cdot \Delta t = \sum_0^{\psi_n} \Delta \frac{u_{rot}^*}{r_{rot}^*} \cdot \Delta \psi \cdot f = 2\pi \equiv 360^\circ \quad (51)$$

Thereby, preferably steps of  $1^\circ$  are made, being finally normalized by the constant  $f$ .

This method shall subsequently be exemplified for the case  $n = 2$ , based on the hitherto derived data, and using the »Excel« computation program of Microsoft-Windows. The units are altogether  $u_1$  for the velocities and  $r_1$  for the radii. The relevant limiting values are:

$$r_{rot,top}^* = 2.00 \quad r_{rot,eq}^* = 1.17158$$

The constant  $f$  has been determined as 2.2618.

**Tab. 2.** Calculation of  $\Psi^*$

$\Psi$ [°]	$\Psi$ [arc]	$\sin(n \cdot \Psi)$	$r_{rot}^*$	$u_{rot}^*$	$\omega^*$	$\bar{\omega}^*$	$\Psi^*$ [arc]	$\Psi^*$ [°]
0	0	0	1.172	0.854	0.729	-	0	0
1	0.017	0.0349	1.173	0.853	0.727	0.728	0.0287	1.6
2	0.035	0.0398	1.176	0.851	0.724	0.725	0.0574	3.3
3	0.052	0.1045	1.181	0.847	0.717	0.720	0.0858	4.9
4	0.070	0.1392	1.188	0.842	0.709	0.713	0.1140	6.5
...	...	...	...	...	...	...	...	...
45	0.785	1	2.000	0.500	0.250	0.250	0.7854	45
...	...	...	...	...	...	...	...	...
90	1.571	0	1.172	0.854	0.729	0.728	1.5708	90
...	...	...	...	...	...	...	...	...
180	3.142	0	1.172	0.854	0.729	0.728	3.1416	180
...	...	...	...	...	...	...	...	...
360	6.283	0	1.172	0.854	0.729	0.728	6.2831	360

Calculation formula for the respective columns:

$$\psi[arc] = \frac{\psi[^\circ]}{180\pi}$$

$$r_{rot}^* = r_{rot,eq}^* + (r_{rot,top}^* - r_{rot,eq}^*) \cdot (\sin(n \cdot \psi))^2$$

$$r_{rot}^* = \frac{u_1 \cdot r_1}{r_{rot}^*}$$

$$\omega^* = \frac{u_{rot}^*}{r_{rot}^*}$$

$$\bar{\omega}^* = \frac{1}{2} (\text{previous} + \text{actual value of } \omega^*)$$

$$\psi^*[arc] = \text{previous value of } \psi^*[arc] + \bar{\omega}^* \cdot f \cdot \Delta\psi[arc]$$

$$\psi^*[^\circ] = \psi^*[arc] \cdot \frac{180}{\pi}$$

Remarkably, at some special points, such as at 0°, 45°, 90°, 180° and 360°,  $\Psi^*$  and  $\Psi$  are equal while normally they differ.

Table 2 represents the basis for all the diagrams comprising the parameter  $\Psi^*$ . Obviously, other variables may then be implemented.

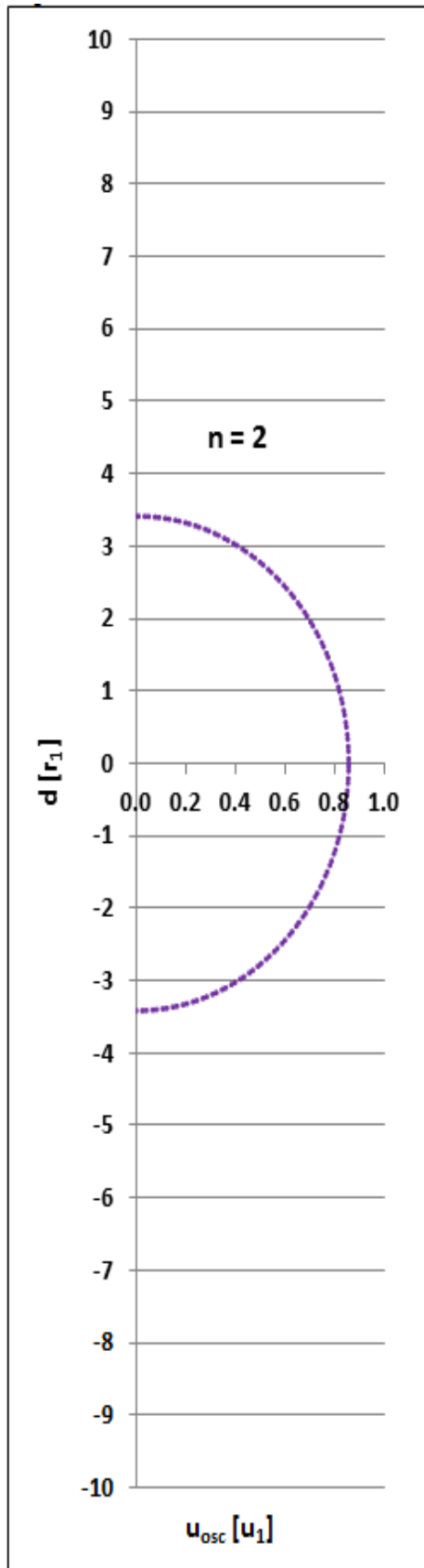


Fig. 4a. Plot of  $u_{osc}$  vs  $d$  for  $n=2$ .

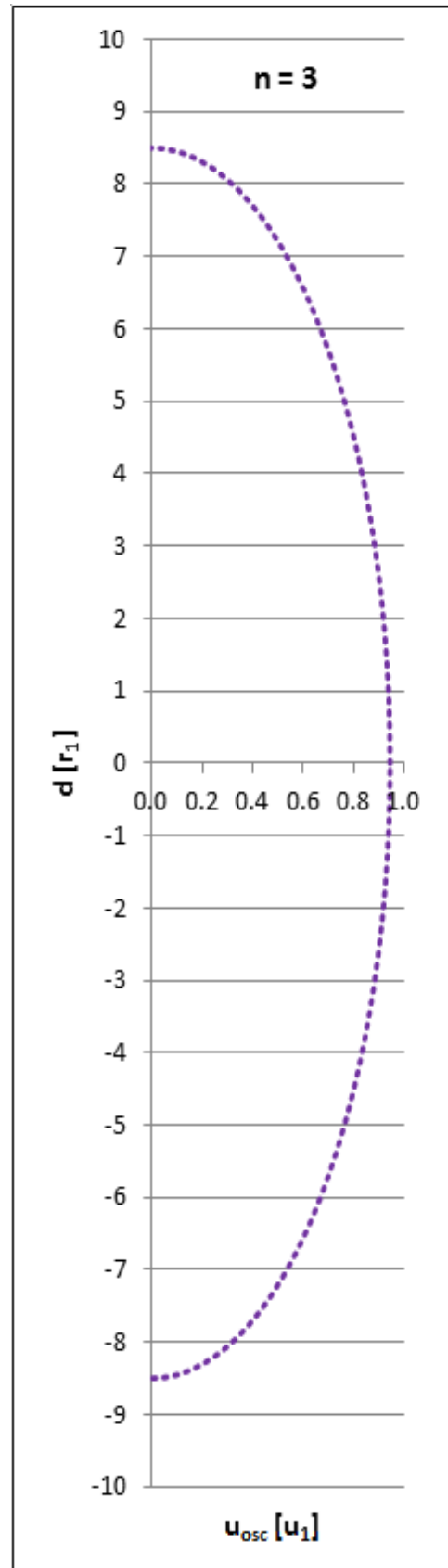


Fig. 4b. Plot of  $u_{osc}$  vs  $d$  for  $n=3$ .

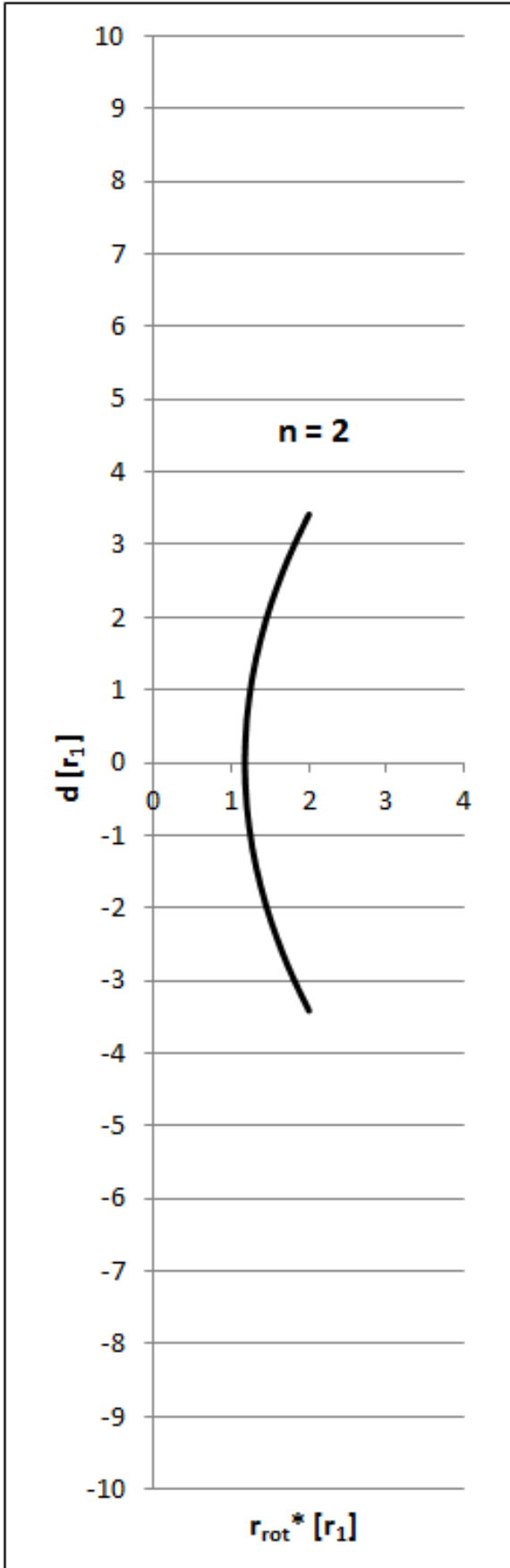


Fig. 5a: Plot of  $r_{rot}^*$  vs  $d$  for  $n=2$ .

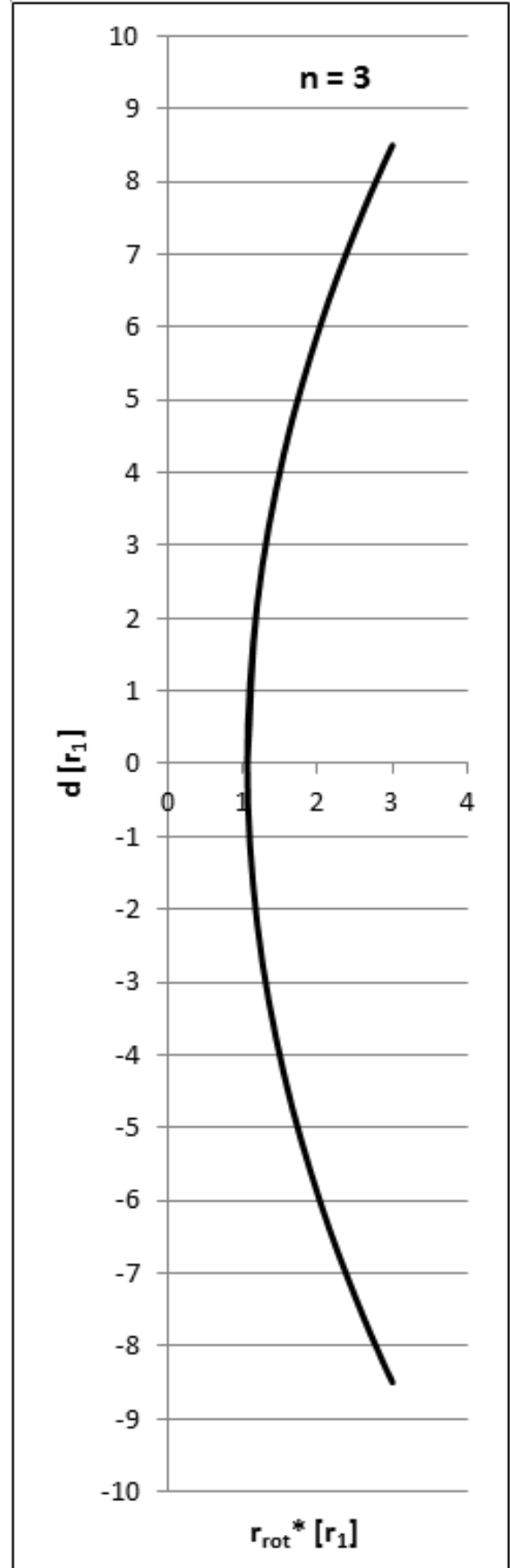


Fig. 5b: Plot of  $r_{rot}^*$  vs  $d$  for  $n=3$ .

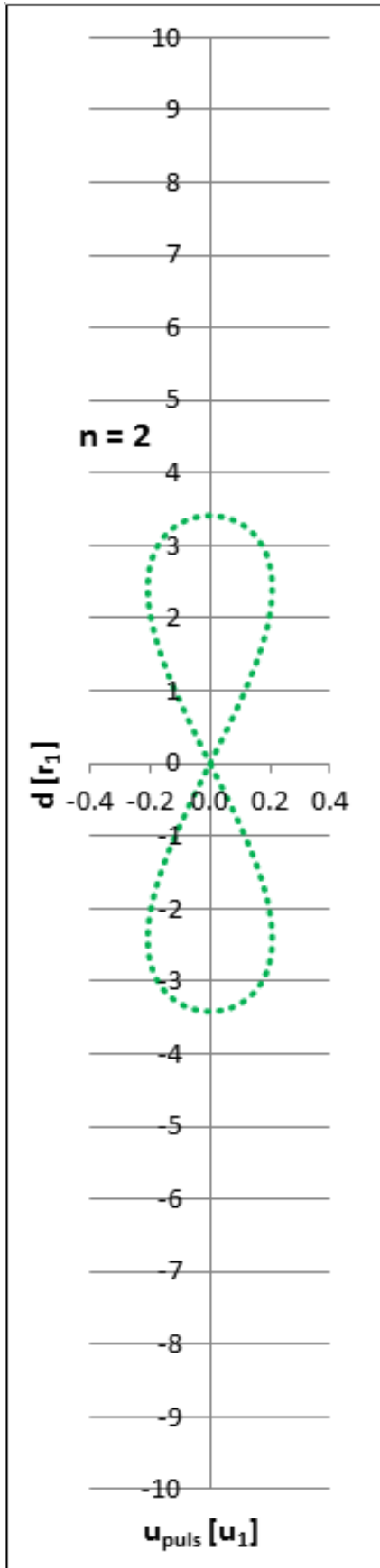


Fig. 6a. Plot of  $u_{puls}$  vs  $d$  for  $n=2$ .

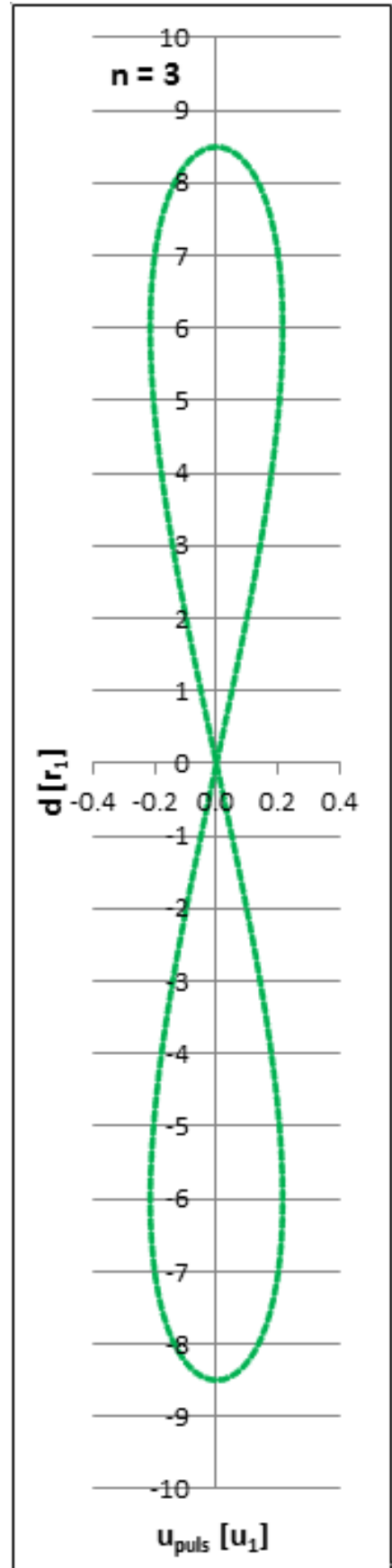


Fig. 6b. Plot of  $u_{puls}$  vs  $d$  for  $n=3$ .

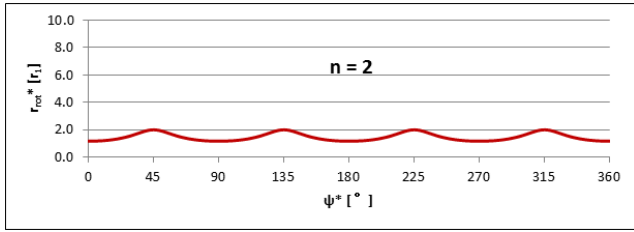


Fig. 7a. Plot of  $r_{rot}^*$  vs  $\psi^*$  for  $n = 2$ .

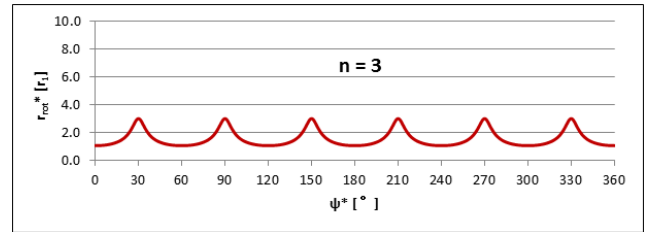


Fig. 7b. Plot of  $r_{rot}^*$  vs  $\psi^*$  for  $n = 3$ .

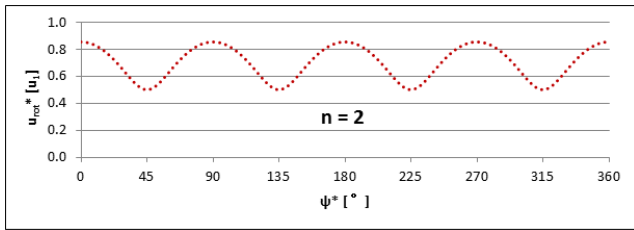


Fig. 8a. Plot of  $u_{rot}^*$  vs  $\psi^*$  for  $n = 2$ .

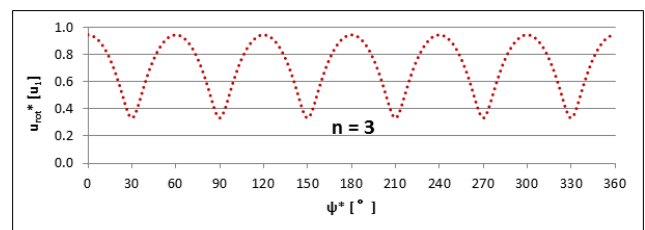


Fig. 8b. Plot of  $u_{rot}^*$  vs  $\psi^*$  for  $n = 3$ .

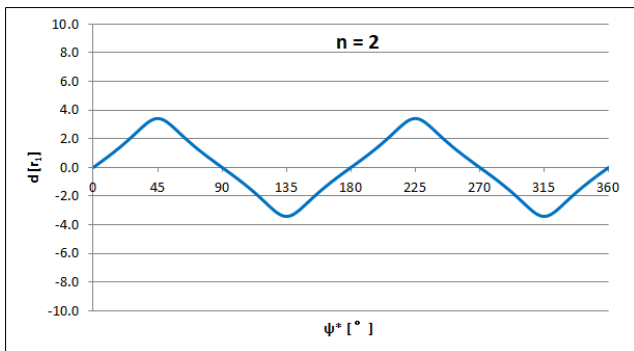


Fig. 9a. Plot of  $d$  vs  $\psi^*$  for  $n = 2$ .

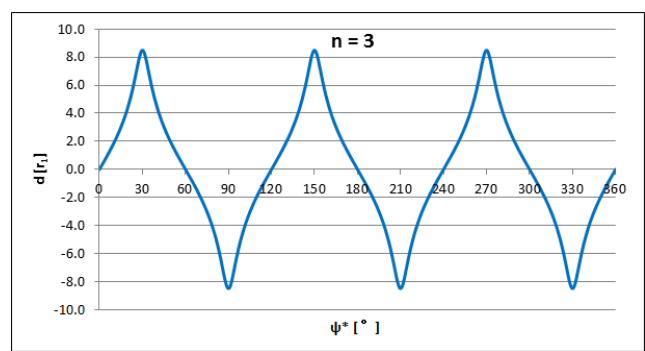


Fig. 9b. Plot of  $d$  vs  $\psi^*$  for  $n = 3$ .

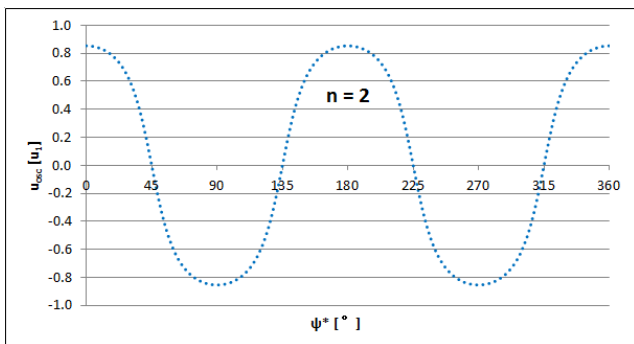


Fig. 10a. Plot of  $u_{osc}$  vs  $\psi^*$  for  $n = 2$ .

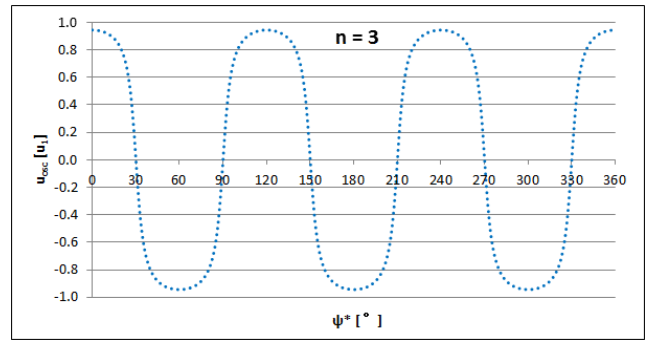


Fig. 10b. Plot of  $u_{osc}$  vs  $\psi^*$  for  $n = 3$ .

## 8. Summary and Conclusions

Starting from the condition for standing waves, it could be derived from the BDB-formulation that a correlation exists between the usual *electron rotation frequency* and a *vibration frequency* being deducible from the De Broglie-wave. Thereby, the two frequencies are coupled to each other via the quantum number  $n$ , being equal when  $n = 1$ . Moreover, it could also be formally shown that a *direct proportionality exists between the light frequency and the vibration frequency of the electron*, which leads to the presumption that

the *vibration* of the electron is the real cause of light radiation as a *resonance effect*, and *not* its rotation around the nucleus. Hence, the usual assumption of a macro-physical radiating Hertz-dipole cannot be maintained. However, this vibration of the electron – i.e. its precise trajectory – could, at first, not really be described.

Nevertheless, subsequently a precise description could be found namely by additionally implementing at least an *oscillation* of the electron being directed *perpendicularly to a virtual rotation plane* and leading to three-dimensional electron trajectories in the excited states while the trajectory in the ground state is planar. This oscillation is accompanied

by a cyclic exchange between potential and kinetic energy. However, it doesn't occur on a cylindrical but on a hyperboloid surface, implicating an additional pulsation. As a consequence, the De Broglie-wave may be considered as the result of a centric electron rotation and an oscillation being induced perpendicularly to the rotation plane and hence *not* influencing the orbital angular momentum at the ground state.

Starting from that assumption, the here proposed hydrogen-model-approach has been made applying solely classical physical regularities, in particular the laws of conservation of energy and of the orbital angular momentum. It has been deduced stepwise and starting from the top and the equatorial position of the electron being assumed as the limiting cases. The necessarily used relation for the harmonic oscillator turned out to be solely approximately but satisfyingly valid. For describing the kinetic energy distribution during the transition states between the limiting cases, the hypothetic relation (48) has been used delivering plausible results. All in all, in spite of the above mentioned approximation concerning the harmonic oscillator, and of the mathematical difficulty of solving the integral (50) explicitly, a quite exact description of the electron trajectories and velocities could be found. Hence, the allegation of the conventional quantum mechanics, according to which an exact description of the electron trajectories would be impossible, is disproved. The thereby underlying misapprehension arises from the application of the probability principle on an isolated particle or particle system, instead of a particle or particle system within an ensemble of particles or particle systems, e.g. being relevant in a thermodynamically describable state.

Beyond that, further effects such as the influence of external magnetic fields on the spectra were beyond the scope of this consideration, even though it's conceivable that the three-dimensionality of the trajectories in the excited states has an influence on the magnetic properties of the atom, while the coherence with the spin seems obvious. Moreover, the existence of a planar and stable ground state exhibiting a definite total energy cannot be explained herewith, solely its principal difference to the wavy and only metastable excited states. And in particular, the ground states of larger atoms, as well as the »Aufbau-principle« of the Periodic Table of the Elements and the occurrence of chemical bonds, are not describable without further ado.

Besides, the final question why the electron doesn't fall down on the nucleus, and that there exists a stable ground state, remains still unanswered. However, a principal explanation may be given by the thought experiment that the electron will not straightway run versus the nucleus when it comes from outside, for its path will be deviated by a magnetic interaction being induced by the electric currents which are engendered by the flying electron as well by the proton. But the relevant computation appears to get quite complex, presumably involving Maxwell's relations, and shall not be tried here.

## Acknowledgement

The present treatise is the result of a more than ten-year avocational work passing through many misperceptions. It would not have been realized without the critical objections of Dr Andreas Rüetschi (physicist, from the Swiss Federal Institute of Technology, Zurich).

## References

- [1] Richard P. Feynman: „QED - The Strange Theory of Light and Matter“, Princeton 1985
- [2] Manjit Kumar: „Quantum, Einstein, Bohr and the Great Debate about the Nature of Reality“, Cambridge 2008. Deutsche Ausgabe: „Quanten“, Berlin 2009
- [3] Johann Jakob Balmer: „Notiz über die Spectrallinien des Wasserstoffs“, *Ann. Phys.* 261/5 (1885), 80-87
- [4] Albert Einstein: „Ueber einen die Erzeugung und Verwandlung des Lichts betreffenden heuristischen Gesichtspunkt“, *Ann. Phys.* 322/6 (1905)
- [5] Niels Bohr: „On the Constitution of Atoms and Molecules“, *Philosoph. Magazine* 26 (1913), 1-25
- [6] Arnold Sommerfeld: „Zur Quantentheorie der Spectrallinien“, *Ann. Phys.* 51 Vol. 356 (1916), 1-96
- [7] H. Mark u. R. Wierl: „Atomformfaktorbestimmung mit Elektronen“, *Z. Phys.* 60 (1930), 741-53
- [8] Gerhard Herzberg: „Atomic Spectra and Atomic Structure“, New York 1944, fig. 15 on p. 31
- [9] Amit Goswami: „Das bewusste Universum“, Lüchow-Verlag, Stuttgart 1997
- [10] Werner Heisenberg: „Über die quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen“, *Z. Phys.* 33 (1925), 879-93
- [11] Erwin Schrödinger: „An undulatory theory of the mechanics of atoms and molecules“, *Phys. Rev.* 28 (1926), 104
- [12] Erwin Schrödinger: „Quantisierung als Eigenwertproblem“ (1. Mitteilung), *Ann. Phys.* 79 Vol. 384/4 (1926), 361
- [13] Erwin Schrödinger: „Quantisierung als Eigenwertproblem“ (2. Mitteilung), *Ann. Phys.* 79 Vol. 384/6 (1926), 32
- [14] Max Born und Pascual Jordan: „Zur Quantenmechanik“, *Z. Phys.* 34/1 (1925), 858-88
- [15] Max Born: „Zur Quantenmechanik der Stossvorgänge“, *Z. Phys.* 37/12 (1926), 863-67
- [16] Detlef Dürr and Stefan Teufel: „Bohemian Mechanics“, Springer 2009
- [17] Dieter Meschede: „Gerthsen Physik“, 21. völlig neu bearbeitete Auflage, Springer 2001